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Experimental certification for nonclassical teleportation

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Summary

How to properly certify the quantum function of a quantum process (device) is one of the key problems in quantum information science, particularly, for a function beyond its classical counterpart. To properly verify quantum teleportation is a critical task because of its fundamental roles in quantum communication and quantum computation. Here, we experimentally certified nonclassical quantum teleportation with one setting by the random teleportation robustness introduced by [Phys. Rev. Lett. 119, 110501 (2017)]. Our results show that the random teleportation robustness is experimentally convenient and powerful for verifying nonclassical teleportation and entanglement.

KEYWORDS

certification, nonclassical teleportation

How to properly verify the function of a quantum device is one of the more increasingly important problems in quantum information science.[1-6] For a general certification protocol,[4,7] we define a proper quantity f to evaluate the quality of the

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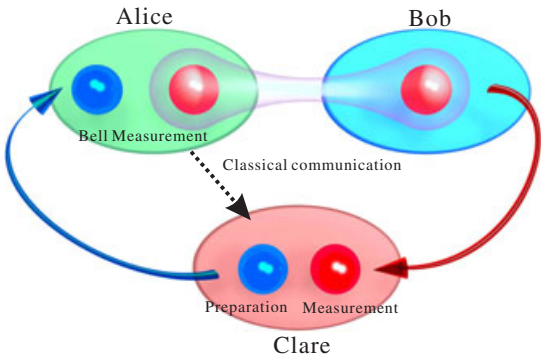

FIGURE 1 Cartoon for the certification process. Alice and Bob share a quantum state and they claim that they can supply a quantum teleportation service between themselves. Clare needs the service and to certify the quality of the teleportation. He sends a set of states (unknown to Alice and Bob) to Alice, who performs a BSM on her side and transmits the results to Clare. At the same time, Bob sends his particles to Clare. Under these conditions, Clare can provide the certification results: accept or reject

quantum device, for example, we can use the fidelity between the experimentally prepared states and the target states to evaluate the quality of the quantum-state-preparation setup. Then, we set a threshold F_0 for the quantity f (the value of F_0 is dependent on the desired task), a maximal failure probability $0 < p < 1$, and a gap δ for the certification process. The test is called a certification test, if it rejects all the devices for which $f < F_0$ and accepts the devices for which $f > F_0 + \delta$ with at least probability $1 - p$.

Quantum teleportation[8-15] is one of the most intriguing applications of quantum mechanics and plays the role of a fundamental tool in quantum communication[16,17] and quantum computation.[18,19] In an ideal quantum teleportation process, Alice and Bob share a perfect Bell state, with Alice capable of a complete Bell-state measurement (BSM) on her side. Then, any unknown state can be transported from Alice to Bob using classical communication regardless of the distance. However, in practice, due to the environment and imperfect control (the state shared by Alice and Bob is generally a mixed state; the measurement of Alice is generally a positive-operator valued measure (POVM), the transportation will be not exact, and the teleportation is not ideal. To certify the quality of the teleportation in a realistic situation as shown in Figure 1 is a key task for the application of quantum teleportation. Particularly, how does one certify nonclassical teleportation?

Here, we experimentally demonstrate the certification of the quality of teleportation, especially certification of nonclassical teleportation, using the random teleportation robustness (RTR) by one-setting partial BSM and the average fidelity (AF) by two-setting BSM.

CERTIFICATION PROTOCOL

There are two different quantities that have been proposed to evaluate the quality of the teleportation: the AF, which is denoted as f[20] (defined in Equation (1)) and the for the experimental data, which is the minimal amount of "white noise" that should be mixed to the experimental data such that the new data can be reproduced by some classical scheme and denoted as r[21] (defined in Equation (2)). The certification protocol used in our experiment is designed as follows: (1) Clare, who will certify the quality of the teleportation between Alice and Bob (who share a quantum state ρ^{AB}), chooses a threshold for the quantity ($F_0 = F_{cl}$, which is the maximal fidelity of the teleportation obtained by Alice and Bob when they share no entanglement for AF, or $R_0 = 0$ for RTR) used to evaluate whether the quality of the teleportation between Alice and Bob matches his requirement, a maximal failed probability $0 < p < 1$, and a gap δ that is correlated with p and the transportation of the error in the experiment. (2) Clare generates a set of quantum states $S = |\phi_i\rangle_C, i = 1, 2, \ldots, m$ (many of the states $|\phi_i\rangle_C$, $i = 1, 2, \ldots, m$ are the same to reduce the statistical error) where $|\phi_i\rangle_C$ is randomly taken from some set of a complete basis of the particle C (the set is denoted as S_0); then, he gives each particle C to Alice. Alice runs a measurement with a POVM element M_a^{AC} (a full POVM for AF and a partial POVM for RTR) on the particles A and C and obtains the result a; she then gives the results to Clare. After the measurement on Alice's side, the particle B belonging to Bob is in the state $\rho^B_{a|i} = tr_{A,C}((M_a^{AC} \otimes I_B)(\rho^{AB} \otimes |\phi_i\rangle_C\langle\phi_i|))$ (unnormalized) with probability $P(a \mid i) = tr(\rho^B_{a|i})$, and Bob also gives this particle to Clare. (3) With the measurement results a from Alice and the particle from Bob, Clare applies a unitary operator U_a (dependent on the results of Alice) on particle B; he then measures particle B. With the complete

BSM on Alice's side, the AF can be defined on the experimentally obtained density matrices $U_a\rho^B_{a|i}U_a^\dagger$ ($a = 1, 2, 3, 4$ and $i = 1, 2, \ldots, m$) by

$$f = \frac{1}{m}\sum_{a,i} P(a|i)\left\langle \phi_i|U_a\rho^B_{a|i}U_a^\dagger|\phi_i\right\rangle. \tag{1}$$

On the contrary, the RTR can be defined on the experimental data, $\rho^B_{a|i}$ ($a = 1, 2$ and $i = 1, 2, \ldots, m$) with a partial BSM on Alice's side (one setting in linear optics), as the solution of the following optimization problem:

$$R\left(\rho^B_{a|i}\right) = \min_{r,\{M_a^{BC}\}} r$$

s.t.

$$\begin{aligned}&\frac{1}{1+r}\rho^B_{a|i} + \frac{r}{1+r}\frac{1}{o_A}\frac{I_B}{d} = tr_C\left(M_a^{BC}(|\phi_i\rangle_C|\phi_i\rangle\langle\phi_i|\otimes I_B)\right)\\ &\forall a, i,\\ &M_a^{BC}\in S \quad \forall a,\\ &\sum_a M_a^{BC} = I_C\otimes\frac{\rho^B + r\frac{I^B}{d}}{1+r},\end{aligned} \tag{2}$$

where o_A is the number of outcomes a (in our partial (complete) BSM, it is 2 (4)), d is the dimension of the system C ($d = 2$ here), and S denotes the set of separable operators. Due to the normalization condition $\sum_a M_a^{AC} = I^{AC}$, the channel operators satisfy $\sum_a M_a^{BC} = I^C\otimes\rho^B$, where ρ^B is Bob's reduced state, which can be seen as a no-signaling condition. The optimal solution r (RTR) gives the minimal amount of "white noise," which is mixed to the experimental data such that the new data admits a classical realization. In addition, the RTR of the given experimental data can be efficiently calculated by the SDP method. (4) Clare compares the quantities (f and r) obtained from the experimental data with the threshold values (R_0 and F_0), respectively. If $r > R_0 + \delta$ ($f > F_0 + \delta$), Clare accepts the teleportation; if $r < R_0$ ($f < F_0$), Clare rejects it.

RANDOM TELEPORTATION ROBUSTNESS

In a standard teleportation process (unnormalized),[21]

$$\begin{aligned}\rho^B_{a|i} &= tr_{AC}\left(\left(M_a^{AC}\otimes I_B\right)\left(\phi_i\rangle_C\langle\phi_i|\otimes\rho^{AB}\right)\right)\\ &= tr_C\left(M_a^{BC}(|\phi_i\rangle_C\langle\phi_i|\otimes I_B)\right),\end{aligned} \tag{3}$$

where

$$M_a^{BC} = tr_A\left(\left(M_a^{AC}\otimes I_B\right)\left(I_C\otimes\rho^{AB}\right)\right). \tag{4}$$

We have established the channel between the input state $|\phi_i\rangle_C$ and the output state $\rho^B_{a|i}$.

The normalization factor $P(a|i) = tr(\rho^B_{a|i})$ is the probability that Alice receives outcomes a given an input state $|\phi_i\rangle$.

Now, we consider that ρ^{AB} is a separable state $\rho^{AB} = \sum_\lambda p_\lambda\rho^A_\lambda\otimes\rho^b_\lambda$. The channel operator M_a^{BC} becomes

$$\begin{aligned}M_a^{BC} &= \sum_\lambda p_\lambda tr_A\left(\left(M_a^{AC}\otimes I_B\right)\left(I_C\otimes\rho^A_\lambda\otimes\rho^B_\lambda\right)\right)\\ &= \sum_\lambda p_\lambda M^C_{a|\lambda}\otimes\rho^B_\lambda,\end{aligned} \tag{5}$$

where $M^C_{a|\lambda} = tr_A(M_a^{AC}(I_C\otimes\rho^A_\lambda))$, and Equation (3) becomes

$$\rho^B_{a|i} = \sum_\lambda p_\lambda tr\left(M^C_{a|\lambda}|\phi_i\rangle_C\langle\phi_i|\right)\rho^B_\lambda. \tag{6}$$

Given the structure of this classical teleportation channel, the RTR for the data can be obtained:

$$
\begin{aligned}
& R\left(\rho_{a|i}^{B}\right)=\min_{r,\left\{M_{a}^{BC}\right\}} r \\
& \text{s.t.} \\
& \frac{1}{1+r}\rho_{a|i}^{B}+\frac{r}{1+r}\frac{1}{o_A}\frac{I_B}{d}=tr_C\left(M_{a}^{BC}(|\phi_i\rangle_C|\phi_i\rangle\langle\phi_i|\otimes I_B)\right) \\
& \forall a,i, \\
& M_{a}^{BC}\in S \quad \forall a, \\
& \sum_{a} M_{a}^{BC}=I_C\otimes\frac{\rho^{B}+r\frac{I^{B}}{d}}{1+r},
\end{aligned}
\tag{7}
$$

where o_A is the number of outcomes a, d is the dimension of the system C, and S denotes the set of separable operators. Due to the normalization condition $\sum_a M_a^{AC}=I^{AC}$, the channel operators satisfy $\sum_a M_a^{BC}=I^C\otimes\rho^B$, where ρ^B is Bob's reduced state, which can be seen as a no-signaling condition. The optimal solution r (RTR) gives the minimal amount of "white noise," which is mixed to the experimental data such that the new data admit a classical realization. We can use the SDP process to calculate the problem.

In the SDP process in the work of Cavalcanti et al,[21] a nonsignaling condition is required. However, considering data fluctuation, the nonsignaling condition cannot be strictly satisfied, and the SDP process cannot be directly applied to experimental data. We must find the optimal approximation that is closest to the nonsignaling condition using the data obtained from the experiment. Similar to our noise model, we assume that Alice's preparation and measurement and Bob's measurement are all perfect, and the fluctuations and imperfections come from the shared channel state ρ_{AB}. We now only have to estimate the optimized channel state ρ_{AB}^{exp} through the experimental results.

We define an estimate function

$$
\Gamma\left(\left\{\rho_{a|i}^{B\text{exp}}\right\}\right)=\sum_{a=1}^{4}\sum_{i=1}^{6}\left(Tr|\rho_{a|i}^{B}-\rho_{a|i}^{B\text{exp}}|\right)^{2},
\tag{8}
$$

where $\rho_{a|i}^{B}$ represents Bob's state obtained from experimental data and $\rho_{a|i}^{B\text{exp}}$ represents the quantum state obtained on Bob's side if we use the determined ρ_{AB}^{exp} as the channel state.

This function represents the gap between the quantum state $\rho_{a|i}^{B}$ we obtain in the experiment and the ideal results using ρ_{AB}^{exp} as the channel state. We define this gap by using the sum of squares of the trace distance for all states. A smaller function value indicates closer agreement with the estimated results and the experimental results. We minimize the value of the function $\Gamma(\{\rho_{a|i}^{B\text{exp}}\})$ via traversing all quantum states and complete the estimation of the entire teleportation process. This process is similar to the maximum likelihood estimation. In this way, the optimized channel quantum state ρ_{AB}^{exp} can be obtained, enabling one to determine the value of RTR.

EXPERIMENTAL REALIZATION OF QUANTUM TELEPORTATION CERTIFICATION

We will experimentally certify the function of a single-qubit-quantum-teleportation–type experimental setup, which is shown in Figure 2. In the experiment, we used two spontaneous parametric down-conversion (SPDC) sources with beam-like phase-matching type-II β-barium borate crystals[22-24] to prepare single-photon (the states generated by Clare) and two-photon states (the states shared by Alice and Bob). The setup of our experiment can be divided into four sections.

In the first section (yellow), we use the SPDC process to produce a single-photon state with a trigger; the state of the single-photon is controlled by Clare, who sends it to Alice. In our experiment, the test states are randomly chosen from the six-quantum-state set: $S_0=|0\rangle,|1\rangle,(|0\rangle\pm|1\rangle)/\sqrt{2}$, and$(|0\rangle\pm i|1\rangle)/\sqrt{2}$, which are prepared by adjusted half-wave plates (HWP1) and quarter-wave plates (QWP1). To reduce the statistical error, each state will be reproduced many times by Clare (he knows which states are the same). In the second section (green), to prepare the state ρ_{AB} that is shared by Alice and Bob, we used a sandwich-like source of entangled photon pairs.[25,26] The quality of the teleportation between Alice and Bob is determined by the form of the state ρ_{AB}. In an ideal teleportation process, we should prepare the maximum entanglement state $|\phi^+\rangle=1/\sqrt{2}(|HH\rangle+|VV\rangle)$. To certify the teleportation with small entanglement, we prepare the state $\rho_{AB}=P|\phi^+\rangle\langle\phi^+|+(1-P)|01\rangle\langle 01|$ where the $|01\rangle\langle 01|$ state is introduced to mix with the maximum entangled state $|\phi^+\rangle$ and the entanglement of the state ρ_{AB} is tuned by the probability P. The PBS1 and HWP2 are used to obtain the

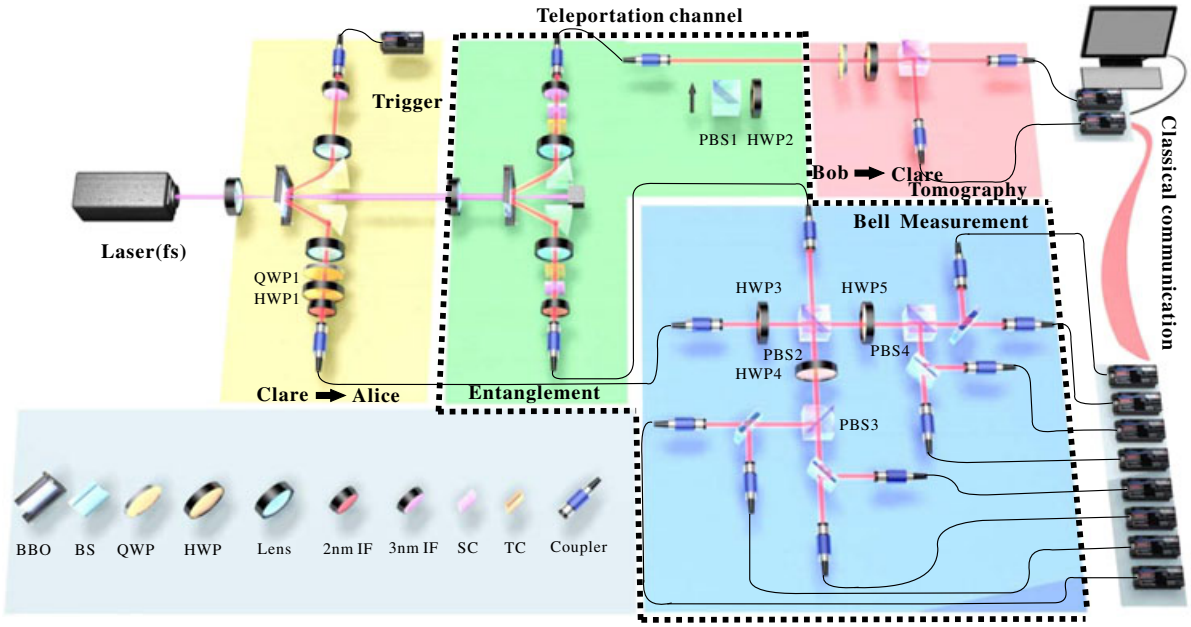

FIGURE 2 Experimental setup. A femtosecond pulse from a mode-locked Ti: Sapphire laser (with a central wavelength of 780 nm, a pulse duration of 90 fs, and a repetition rate of 76 MHz) first passes through a frequency doubler. The emitted ultraviolet pulses are then used to pump two type-II spontaneous parametric down-conversion (SPDC) sources. The first SPDC source produces a product state $|H\rangle \otimes |V\rangle$. One of the photons is used to prepare the state $|\phi_i\rangle_C$ by QWP1 and HWP1. Another photon is used as the trigger for the first photon. The second SPDC source produces a maximally entangled state. The compensation crystals are used to compensate the spatial and temporal differences between the two orthogonal polarizations. PBS1 and HWP2 are used to prepare the $|H\rangle \otimes |V\rangle$ state and a classical mixture of the maximum entangled states that we have prepared to realize $P|\phi^+\rangle\langle\phi^+| + (1-P)|01\rangle\langle 01|$. PBS2-4, HWP3-5, and BSs constitute an improved Bell measuring device, particularly, the partial Bell-state measurement $M_a^{AC}(\{|\phi^+\rangle, I - |\phi^+\rangle\})$ can be realized by this device, and the measurement of four Bell states can be achieved by rotating HWP3. At the two inputs of the Bell measurement device, we use a 2-nm interference filter to improve the visibility of the Hong-Ou-Mandel interference between the two independent sources. Finally, a set of standard single-qubit tomography devices exist on Clare's side to reconstruct the $\rho_{a|i}^B$ with the classical information obtained by the M_a^{AC} measurement. BBO, barium borate; BS, beam splitters; HWP, half-wave plates; PBS, polarizing beam splitters; QWP, quarter-wave plates

state $|01\rangle\langle 01|$. By controlling the inserted time for the PBS1 and HWP2 in the light path with a certain proportion, we can obtain the state ρ_{AB}. The third section (blue) is a complete (partial) linear optical BSM (corresponding to the measurement M_a^{AC} on Alice's side on particles A and C) device for AF (RTR), which consists of polarizing beam splitters (PBS2-4), half-wave plates (HWP3-5), beam splitters (BSs), and single-photon detectors. Due to the limitation of linear optics, only two BSMs ($|\phi^\pm\rangle$ or $|\psi^\pm\rangle$) can be carried out at the same time. There are two settings of BSM needed to obtain AF. Four Bell states are measured by rotating HWP3 to 0° or 45°. However, the RTR can be determined by only one-setting partial Bell-state measurements on $\{|\phi^+\rangle, I - |\phi^+\rangle\}$. Unlike the standard complete BSM device, the partial BSM generally needs a photon-number resolving detector. In our experiments, the partial BSM is realized in another way to avoid the poor performance of the photon-number resolving detector.[27] Beam splitters are added to each outlet, and a beam of light is divided into two beams; as a result, we have 50% probability to distinguish the situation in which two photons appearing on the same paths from others, and the partial BSM is achieved. In principle, the successful probability can approach 100% by adding enough BSs. The measured results are sent to Clare through classical communication. The second section and the third section constitute the teleportation channel, with Clare wanting to certify the quality of the teleportation channel. The fourth section (pink) is that used to send Clare the classical information from Alice, including a standard single-qubit state tomography device to obtain the state $\rho_{a|i}^B$.

EXPERIMENTAL RESULT

First, we consider the case of ideal quantum teleportation. For this purpose, we use the setup in the second section in Figure 2 to prepare a maximum entangled state $\rho_{AB} = |\phi^+\rangle$. In our experiment, the prepared state is close to the ideal

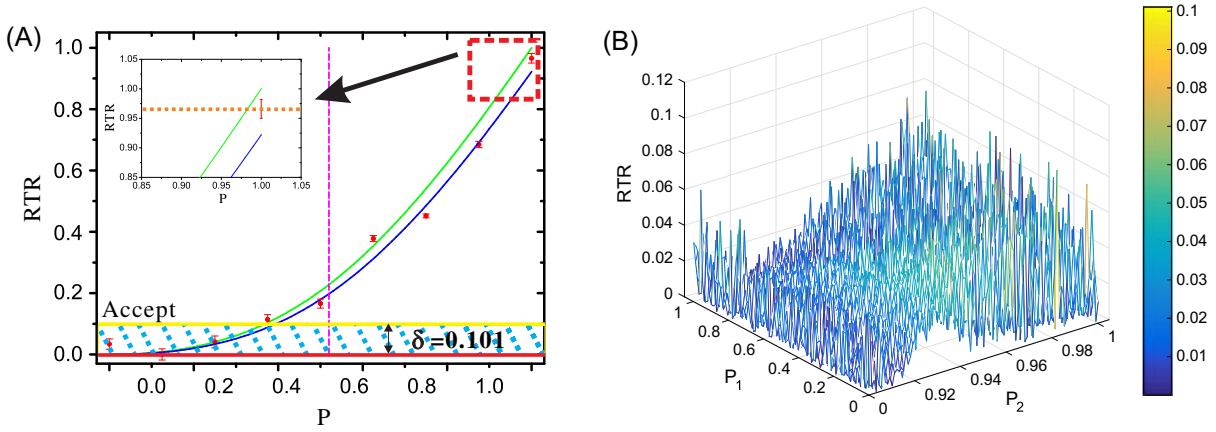

FIGURE 3 Certification for teleportation via random teleportation robustness (RTR). A, Certificate the nonclassical teleportation of the states $\rho_{AB} = P|\phi^+\rangle\langle\phi^+| + (1-P)|01\rangle\langle 01|$ by RTR. The green solid line is the theoretical curve for the RTR as a function of P (without noise). The blue solid line is obtained from our noise model. The purple dotted line corresponds to the state with AF= $F_{cl} = 0.667$, and its left states cannot be certified by AF. The red dots represent the experimental results and the only statistical error considered in the error bar. In this certification test, the threshold of the RTR is set as $R_{cl} = 0$, the fail probability is 1/10 000, and $\delta = 0.101$. With these parameters, the dots above the yellow lines (RTR= 0.101) are accepted and no dots with an entangled channel are directly rejected. The gap δ (with fail probability 1/10 000) are estimated by Monte Carlo method. B, We use the Monte Carlo method to simulate the transportation of the error and the relation between the gap δ and the maximal fail probability. The simulation is based on the noise model (ie, the shared states in the form $\rho^{AB}_{\text{noise}} = P_2(P_1\phi^+ + (1-P_1)|01\rangle\langle 01|) + (1-P_2)I$) and the Poisson distribution, and 10 000 samples are randomly generated. The biggest error δ is 0.101

state with a fidelity $F = 0.990 \pm 0.003$. The fidelity of the BSM was improved by using single-mode fibers and narrowband frequency filters (full width at half maximum of 2 nm) ensuring high-quality Hong-Ou-Mandel (HOM) interference. We measured a resultant HOM visibility of $V = 0.971 \pm 0.006$. The whole experimental four-fold coincidence counting rate is 1.25 coincidences/s. To reduce the statistical error for the data, each data point is calculated using approximate 36 000 detected events. The AF for the state set S_0 obtained in our experiment is 0.971 ± 0.003. This is the best quality obtained for quantum teleportation in our experiment.

The difference between our experimental AF and the ideal AF is mainly due to imperfection in the quantum state ρ_{AB}. The errors in the state ρ_{AB} are experimentally induced by two resources: one is the imperfect interference in the teleportation process, and the other is the imperfection of polarization devices, such as PBSs and HWPs. Based on these imperfections, we can establish a noise model for the quantum state ρ_{AB} (see Noise Model). The theoretical results based on this noise model are also shown in Figure 4a, which agree well with the experimental results.

It is more difficult to obtain the RTR from experimental data due to the nonsignal condition. Generally, the experimental data cannot exactly satisfy this requirement; however, the nonsignal condition of the experimental data in the optimization problem is necessary. To solve this problem, based on the former noise model where we assume that both the single-qubit preparation and measurement are perfect, the experimental data are fitted by the optimal quantum state ρ_{AB}. All the RTRs are obtained based on the optimal ρ_{AB}.

Generally, to make the previous protocol a certification test, we should carefully consider the relation between the probability p, the errors in the experiment and the gap δ. Practically, based on the noise model and the Poisson distribution, we use the Monte Carlo method[28] to simulate the transportation of the error and the relation between the gap δ and the maximal fail probability p (see Figure 3b). We simulate 10 000 samples and set the largest deviation (0.101 for RTR and 0.034 for AF) as the gap δ; thus, when the practical RTR is larger than 0.101 (AF larger than $F_{cl}+0.034$), the device demonstrates the nonclassical teleportation at least with probability 1/10 000. With these parameters, the previous protocol is a certification test for nonclassical teleportation.

For near-perfect quantum teleportation, both quantities can be used to certify this type of teleportation very well. However, AF and RTR show different behaviors when they are used to certify the nonclassicality of the teleportation in which Alice and Bob share a quantum state with small entanglement. It has been proven that there exists a situation where Alice and Bob can share states with entanglement (such as bound entangled states), which, however, can never be accepted by the certification with quantity AF. However, if we use the RTR as a quantity to evaluate the quality of the

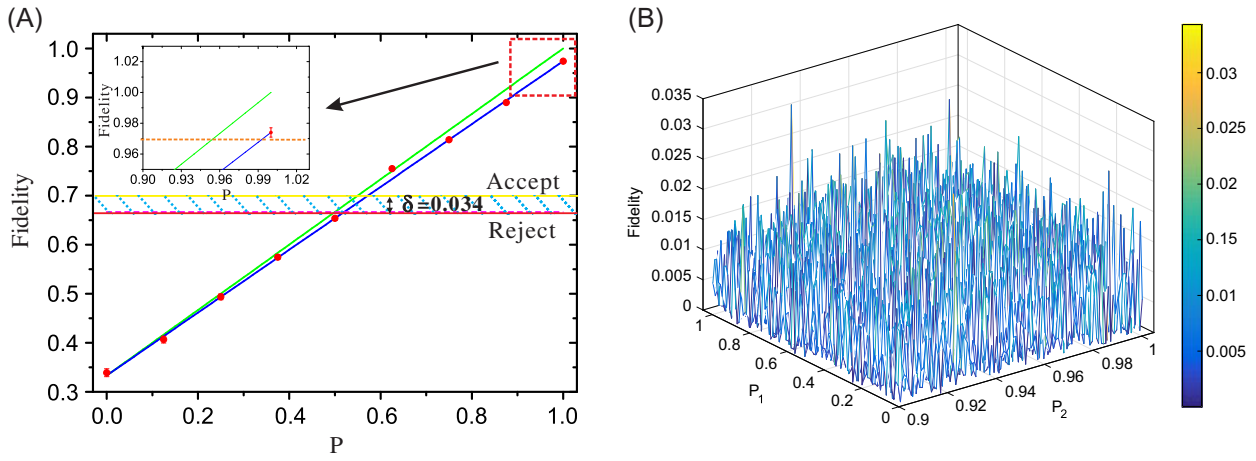

FIGURE 4 Certification for teleportation via average fidelity (AF). A, The green line represents the AF when Alice and Bob share the ideal state $\rho_{AB} = P|\phi^+\rangle\langle\phi^+| + (1-P)|01\rangle\langle 01|$. The blue line represents the theoretical results estimated by our noise model, whereas the red points show our experimental values. In this certification test, the threshold F_{cl} is set as 0.666, the fail probability is 1/10 000, and the gap $\delta = 0.034$. The relation between the fail probability and the gap are simulated by the Monte Carlo method. The system error was estimated by a Monte Carlo method, ie, $\delta = 0.034$. Based on these parameters, the dots ($P = 0.625, 0.75, 0.875, 1$) above the yellow line (AF= 0.7) are acceptable; the dots ($P = 0, 0.125, 0.25, 0.375, 0.5$) below the red line (AF= F_{cl}) are directly rejected. B, We use the Monte Carlo method to simulate the transportation of the error and the relation between the gap δ and the maximal fail probability. The simulation is based on the noise model (ie, the shared states in the form $\rho^{AB}_{\text{noise}} = P_2(P_1\phi^+ + (1-P_1)|01\rangle\langle 01|) + (1-P_2)I$) and the Poisson distribution, and 10 000 samples are randomly generated. The biggest error δ is 0.034

teleportation, ideally, all ideal teleportation involving Alice and Bob sharing an entangled state can be demonstrated to show nonclassical character.

Now, we turn to demonstrate the key difference between these two different certifications. We prepare the states shared by Alice and Bob as $\rho_{AB} = P|\phi^+\rangle\langle\phi^+| + (1-P)|01\rangle\langle 01|$ with different $P = \{0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1\}$, and the teleportation between Alice and Bob are based on these states.

The RTR and the AF obtained by the experimental data are shown in Figure 3a and Figure 4a. It is clear that Clare accepts the nonclassical teleportation when $P = 0.375, 0.5, 0.625, 0.75, 0.875, 1$, no results are directly rejected by using RTR. Meanwhile, Clare accepts the nonclassical teleportation when $P = 0.625, 0.75, 0.875, 1$ and rejects when $P = 0, 0.125, 0.25, 0.375, 0.5$ by using AF.

NOISE MODEL

The AF is 1 when one use a maximally entangled state as the shared state, whereas in our experiment, the measured AF is 0.971, which is slightly lower than the theoretical prediction. This is due to imperfection in state preparation, measurement, and interference. In our experiment, we consider that all the imperfections come from the imperfect preparation of the shared channel state ρ_{AB} because Alice's state preparation and measurement and Bob's measurement are nearly perfect. The imperfection in ρ_{AB} comes from two parts.

One part is distinguishability for the two-photon pairs from two down-conversion processes. This imperfection can be revealed by the interference visibility on the basis of $(H+V)/\sqrt{2}, (H-V)/\sqrt{2}$ or $(H+iV)/\sqrt{2}, (H-iV)/\sqrt{2}$. This noise can be simplified to an incoherent item $|HH\rangle\langle HH| + |VV\rangle\langle VV|$.

The other part comes from the classical items, which are due to the imperfect polarization devices PBSs, HWPs, and so on. This will decrease the extinction ratio on the basis H/V. In our experiment, this noise is much smaller than the first one. In our experiment, this noise will introduce another item $|HV\rangle\langle HV| + |VH\rangle\langle VH|$. Therefore, we can get our noise model and rewrite the shared channel state as

$$\begin{aligned}\rho^{\text{noise}}_{AB} = P(p_1\phi^+ + p_2(|HH\rangle\langle HH| + |VV\rangle\langle VV|) \\ + p_3(|HV\rangle\langle HV| + |VH\rangle\langle VH|)) + (1-P)|HV\rangle\langle HV|.\end{aligned} \tag{9}$$

TABLE 1 The fidelity for the teleported states

$F(\omega_x, \sigma^B_{a\|\omega_x})$	0	1	2	3	4	5
0	0.963 ± 0.019	0.942 ± 0.019	0.927 ± 0.019	0.997 ± 0.007	0.999 ± 0.009	0.989 ± 0.014
1	0.978 ± 0.015	0.932 ± 0.019	0.986 ± 0.014	0.957 ± 0.018	0.990 ± 0.012	0.996 ± 0.008
2	0.991 ± 0.012	0.986 ± 0.011	0.910 ± 0.020	0.966 ± 0.014	0.972 ± 0.016	0.992 ± 0.012
3	0.946 ± 0.019	0.962 ± 0.018	0.986 ± 0.009	0.953 ± 0.019	0.995 ± 0.016	0.989 ± 0.013

In our experiment, the measured visibility on the basis $(H+V)/\sqrt{2}, (H-V)/\sqrt{2}$ is 0.928 and, on the basis of H/V, is 0.987. According to these two values, we can estimate the form of our channel state ρ^{noise}_{AB}

$$\rho^{\text{noise}}_{AB} = P \begin{pmatrix} 0.497 & 0 & 0 & 0.464 \\ 0 & 0.003 & 0 & 0 \\ 0 & 0 & 0.003 & 0 \\ 0.464 & 0 & 0 & 0.497 \end{pmatrix} + (1-P) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

The fidelity for the teleported states is shown in Table 1; here, we only consider the statistical errors.

ERROR ESTIMATION

We use a Monte Carlo method to estimate our whole RTR error. The two process is calculated. In the first process, we use our existing noise model to assume a number of shared ideal quantum states $\rho^{AB}_{\text{noise}} = P_2(P_1\phi^+ + (1-P_1)|01\rangle\langle 01|) + (1-P_2)I$. Using these ideal quantum states, we obtain theoretical $\text{RTR}_{\text{ideal}}$.

In the second process, we use this ideal shared state ρ_{AB} to deduce the ideal state $\sigma^B_{A|i}$ obtained by Bob. Through these states, Poisson distribution can be used to complete the simulation of experimental $\sigma^{B'}_{A|i}$. Then, the maximum likelihood estimation method is used to estimate the closest shared quantum state ρ^{AB}_{noise}. Finally, we get the simulated $\text{RTR}_{\text{simulation}}$.

We analyze the experimental error and certification process by the difference($\delta = |\text{RTR}_{\text{ideal}} - \text{RTR}_{\text{simulation}}|$) between the RTR simulated by poisson distribution and the RTR in theory. We use the noise model to select 10 000 states evenly and use Poisson distribution to simulate each teleportation process. After 10 000 random sampling (Figure 3b), we find the upper bound of the confidence interval as the maximum value here.

$$\begin{aligned} &\text{Ideal Process} \\ &\rho^{AB}_{\text{noise}} \to \left\{\rho^B_{a|i}\right\} \to \text{RTR}_{\text{ideal}}, \\ &\text{Monte Carlo Process} \\ &\rho^{AB}_{\text{noise}} \to \left\{\rho^B_{a|i}\right\} \overset{\text{Poisson}}{\Rightarrow} \overset{\text{distribution}}{} \left\{\rho^{B'}_{a|i}\right\} \\ &\overset{\text{Estimate}}{\Rightarrow} \rho^{AB'}_{\text{noise}} \to \text{RTR}_{\text{simulation}}. \end{aligned} \tag{11}$$

The estimation of fidelity error is similar to the RTR error. The only difference is that there is a lack of estimation process. The simulation results are shown in Figure 4b.

$$\begin{aligned} &\text{Ideal Process} \\ &\rho^{AB}_{\text{noise}} \to \left\{\rho^B_{a|i}\right\} \to \text{AF}_{\text{ideal}}, \\ &\text{Monte Carlo Process} \\ &\rho^{AB}_{\text{noise}} \to \left\{\rho^B_{a|i}\right\} \overset{\text{Poisson}}{\Rightarrow} \overset{\text{distribution}}{} \left\{\rho^{B'}_{a|i}\right\} \\ &\to \text{AF}_{\text{simulation}}. \end{aligned} \tag{12}$$

CONCLUSION

In summary, we have experimentally certified the quality of quantum teleportation by two different quantification metrics: AF and RTR. Our results show that RTR is more powerful than AF. Some quantum states that cannot demonstrate nonclassical teleportation by AF can be certified by RTR. Moreover, RTR can be obtained by a partial BSM and is more friendly in experiment. Furthermore, due to the relationship between the entanglement of state ρ_{AB} and nonclassical teleportation between Alice and Bob certified by RTR, the RTR certification here can also be directly viewed as the certification of quantum entanglement.

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